Lemke-Hobson Example

Monday, 29 August 2022 4:50 PM

Let $R = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 2 & 2 & 2 \end{bmatrix}$ be a symmetric game.

Polytope:

$x_1 \geq 0$

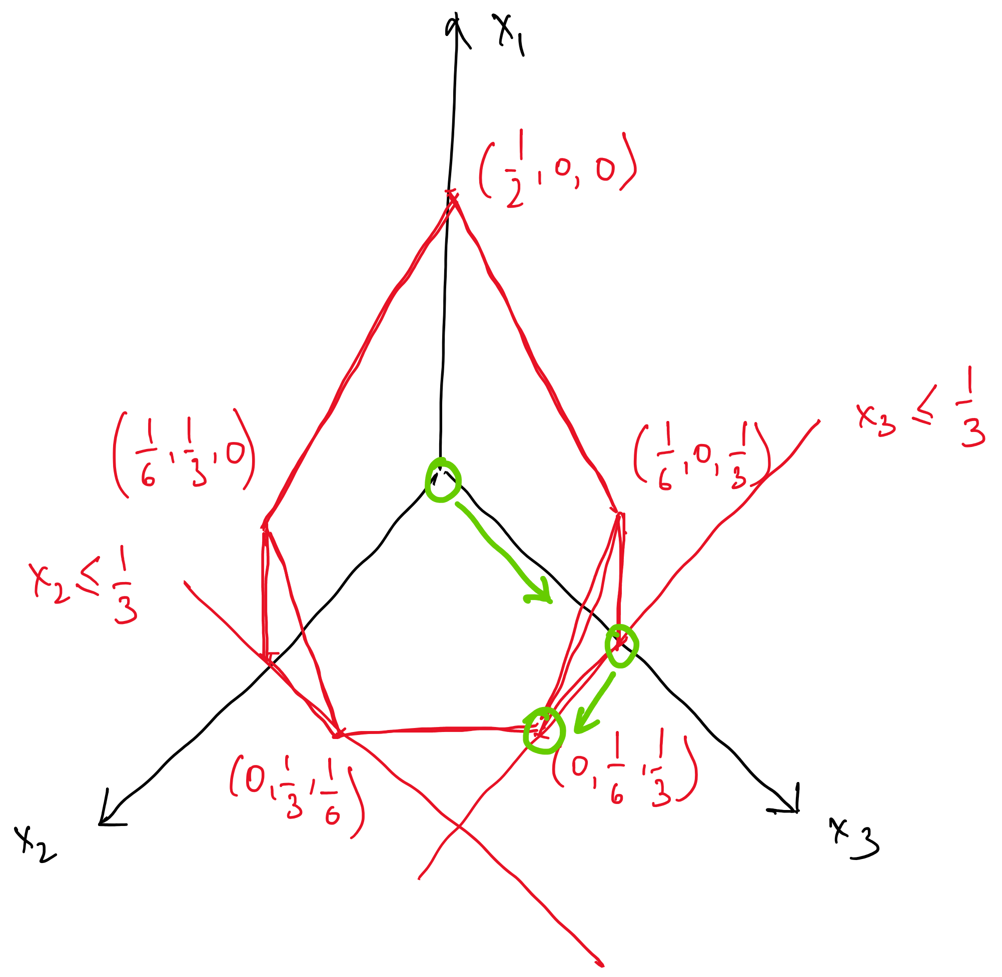
$x_2 \geq 0$

$x_3 \geq 0$

$3x_2 \leq 1$

$3x_3 \leq 1$

$2x_1 + 2x_2 + 2x_3 \leq 1$



Then $v_0 = (0, 0, 0)$. Tight constraints: $x_1 = 0$, $x_2 = 0$, $x_3 = 0$.

① Untighten $x_3 = 0$. Find z: $\left.\begin{array}{l} z_1 = 0 \\ z_2 = 0 \\ z_3 = 1 \end{array}\right\} (0, 0, 1)$

Then increasing λ, $x + \lambda z \geq 0 \quad \forall \lambda \geq 0$

$R(x + \lambda z) = \begin{bmatrix} 0 \\ 3\lambda \\ 2\lambda \end{bmatrix} \leq 1, \Rightarrow \lambda \leq \frac{1}{3}$

Thus, next vertex is $(0, 0, \frac{1}{3})$, tight constraints: $x_1 = 0$, $x_2 = 0$, $(Rx)_2 = 1$

Now coordinate 2 is represented twice, & $x_2 = 0$ was tight earlier, so we untighten $x_2 = 0$.

Find z: $\left.\begin{array}{l} z_1 = 0 \\ 3z_3 = 0 \\ z_2 = 1 \end{array}\right\} (0, 1, 0)$

Again, $x = (0, 0, \frac{1}{3}) + \lambda z$ is feasible for all $\lambda \geq 0$

$R(x + \lambda z) = \begin{bmatrix} 0 \\ 1 \\ 2/3 \end{bmatrix} + \begin{bmatrix} 3\lambda \\ 0 \\ 2\lambda \end{bmatrix} \leq 1$

$\Rightarrow \begin{bmatrix} 3\lambda \\ 0 \\ 2\lambda \end{bmatrix} \leq \begin{bmatrix} 1 \\ 0 \\ 1/3 \end{bmatrix} \Rightarrow \lambda \leq \frac{1}{6}$

Thus next vertex is $(0, 1/6, 1/3)$, tight constraints are $x_1 = 0$, $(Rx_2) = 1$, $(Rx_3) = 1$

This is a democracy! hence scaling up, $(0, 1/3, 2/3)$ is an equilibrium.